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The Instantaneous Approximation to the Transition Matrix Elements between Two Bound States [1]

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Abstract

Under the framework of the Bethe-Salpeter (B.S.) wave functions and the Mandelstam formalism as well, to make "instantaneous approximation" to a transition matrix element (a current operator sandwiched between two bound-states of double heavy quarks) is described. By taking the typical concerned decays as examples, such as B_c meson decaying to $J/\psi + (\bar{l}\nu)$, the advantages of the approach and its limitations are illustrated. Finally, potential applications to various processes for possible double heavy flavoured systems, such as those of $(Q'\bar{Q})$ and $(Q'Q)$ $(m_Q, m_{Q'} \gg \Lambda_{QCD})$, are discussed.

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The Standard Model (S.M.), as well-known, has made great successful. The flavour decays i.e. the weak decays, of the fundamental fermions (quarks and leptons) are supposed to be understood very well in the framework of the S.M. i.e. the vector boson exchange and the CKM (Cabibbo-Kobayashi-Maskawa) mixing matrix structure. However, except the leptons and the undiscovered top quark, all the known quarks always decay as constituenets of hadrons, once they are produced. This is due to the fact that colour has confinement nature and the quarks carry colour explicitly, thus a quark always 'fragment' into hadrons before decaying once it is produced. (Only the undiscovered top quark may be an except, as its mass is very great hence its lifetime is so short that there is no time to fragment into hadrons before decaying.) Thus we always see flavour decays through hadrons in experiments and we have to deal with the effects of the confinement when studying the flavour decays of quarks. However, the confinement effects are of a nonperturbative nature so very difficult to compute them reliablely. Hence the weak decay problem has a very long story, which had started before the S.M. was established. Nevertheless the progresses on the problem have always being achieved steadly, especially the recent achievements of the Heavy Effective Theory (HET) for one (or two) light and one heavy flavour hadrons: $Q\bar{q}$ or Qqq ($m_Q >> m_q$) and their antiparticles. Due to the authors of HET[1−6], a great progress has been achieved in understanding the weak decay of heavy flavours under the approximation $m_Q \to \infty$ and with the corrections of the order $1/m_Q$ and higher. The authors note at very beginning when they proposed the approach that one of the applicable conditions of their formalism is that the mass of the heavy quark in the concerned hadrons should be greater than Λ_{QCD} and much greater than that of the light one.

In the meantime as emphasized by many authors[7-11], the B_c meson will play a special role for understanding the heavy flavour decays and some progresses have achieved too.

The B_c meson is the ground state of the bound states for the system of the heavy quark-antiquark pair c and $\bar{b}$. Both of the quarks $\bar{b}$ and c are heavy quarks i.e. $m_Q \ll \Lambda_{QCD}, Q = b, c$. Though both may be considered being nonrelativistic, their masses are compatible, thus the formalism[1−6] may not be very appropriate, at least careful examination are needed and considerable modifications are expected.

Of the three generations in the S.M., besides the well-studied systems of the $b\bar{b}$ and the $c\bar{c}$, the quark-antiquark c and $\bar{b}$ and their antiparticles are the only ones which still have a long enough lifetime to form a bound state before one of them decaying. In contrary, the rest possible heavy quark pair systems must contain a top or anti-top quark at least. They will not have chance to form a bound state. It is due to a great mass of the top quark (the top mass m_t greater than $140GeV$[12], though the top has not discovered yet) that causes a great phase space so a very short lifetime even in the case only weak decays are allowed, hence there is no time to form a bound state before the top decaying. The c and $\bar{b}$ system so interesting is attributed to the fact that it is the only possible one which may make mesons and the mesons have not been discovered yet. Besides it plays a special role in understanding the heavy flavour decays. The B_c meson, the ground state of the c and $\bar{b}$ system, not as that of the c(or b) and $\bar{c}(\bar{b})$ systems, carries flavors explicitly so no strong decay at all i.e. the ground state is stable to the strong interaction. It has a great advantage for studying the weak decay and the bound state effects of a heavy quark-antiquark system, even for the later only, still better than those of the J/ψ and Υ families. Indeed, the ground state, the B_c meson, has many sizable and various weak decay channels, so that it becomes possible to compare the results of experiments and theories on the weak interaction effects and the binding effects of a heavy quark pair system widely.

Usually the above two kinds of effects in the meson decays are divided into two categories from another aspect for convenience: the short distance effects i.e.

the weak interaction responding for the flavour change and its perturbative QCD corrections; and the long distance ones i.e. the binding effects for the components. In general speaking, although they are entangled, the second ones, being of non-perturbative nature, are very difficult to deal with aliably, while the former ones, being perturbative nature, are relatively easy to treat by means of the Standard Model of electro-weak interaction and perturbative QCD up to a desired accuracy. However, for the non-perturbative binding effects of a heavy quark system, great progresses have been achieved under the framework of the QCD inspired potential model, but in some phenomenological sense, especially for the J/ψ and Υ families. Thus we may consult the potential model to treat the binding effects for the c and $\bar{b}$ system too. According to the QCD inspired potential model, the B_c meson is very similar to the two families, of J/ψ and Υ, but has a different reduce mass only. Thus the Shrödinger equation with the same QCD inspired potential and a corresponding different reduce mass, should be worked very well to depict the bending effects of the c and $\bar{b}$ system. The Shrödinger equation, as known being in a sense of nonrelativistics, hence to the static properties of the system, is good enough, while to the decays, especially those with a great recoil momentum may be problematic when using the solution(s) as the wave function(s) directly. To find a way to use the Schödinger wave function(s) properly into the decays of the double heavy quark systems is the aim of the paper.

To our knowledge, the Mandelstam formalism[13] with the Bethe-Salpeter (B.S.) wave functions is one of the best approaches to compute a matrix element sandwiched between two single bound states, however which is very often to be attributed at last when calculating a whole decay matrix element. Due to Salpeter[14], the relation of a B.S. wave function to the Shrödinger one was established quite long time ago by the so-called instantaneous approximation. Therefore it seems no problem for the present purpose that the Mandelstam formalism might be used with the help

of the Shrödinger wave functions by solving the reduced and corresponding nonrelativistic equation. Nevertheless, in the Salpeter's approach, the approximation taken at the centre mass system of the bound state is implicated. If only to apply the Shrödinger wave functions to calculating those decay matrix elements sandwiched between two states, but of which only one bound state is involved, it is straightforward relatively, provided one calculates such a matrix element at the centre mass system of the bound state. However now for the most important decays of the B_c meson, the problem is not so straightforward because the decay matrix element is attributed to calculate a weak current matrix element with quite a great momentum recoil, the current operator sandwiched between two bound state. In this paper we are generalizing the instantaneous approximation to suit the interesting cases.

To be the start point, we suppose that the transition matrix elements of the B_c meson, sandwiched between two single bound states, (the form factors or the decay current matrix elements) may be computed in a desired accuracy with the help of the Mandelstam formalism no matter how great the momentum recoil, provided that the one-particle irreducible green function, responsible for the matrix element in the formalism, matches to the trancated kernel of the B.S. equation well, as required by the formalism[15]. It is because the formalism is fully relativistic. Now the problem is attributed to having reliable B.S. wave functions for the quark bound states, appearing in the formalism. In general as for hadrons the key problem to obtain reliable B.S. wave functions is to establish the B.S. equation with a reliable kernel and to find a suitable method to solve the equation. However the nonperturbative nature of the binding effects between the quarks from the QCD confinement makes the problem very difficult. For those matrix elements sandwiched between two double heavy quark bound state(s) such as the B_c meson and J/ψ (or Υ) etc. the B.S. wave functions occurring in the formalism can be acquired reliablly by means of the instantaneous approximation, i.e. the B.S. kernel for the double heavy quark

systems may be related to the QCD inspired potential of the Schödinger equation esteblished in a potential model framework, so the B.S.'s and Schrödinger's wave functions are related too through the approximation. Whereas one could not set the initial and final states in a reference frame both at rest for a real decay process, especially when a matrix element with quite a great momentum recoil is considered, thus the relation between the wave functions of the potential model and the B.S. ones, established only at a rest frame (C.M.S.) as indicated by the original instantaneous approximation, still cannot be applied straightforward. Here we constrain ourselves to consider the transitions between the states of double heavy quark systems. The strategy, we adopt is to generalize the instantaneous approximation to apply to the whole matrix elements no matter how great the momentum recoil. We note here that as the wave functions are computed out only numerically in the potential model, one cannot use the method by Lorentz boosting it directly to pursue the present purpose reliably. Our approach here, essentially to say, is to calculate the matrix elements under the Mandelstam formalism and to apply the 'instantaneous approximation' to the matrix elements in a whole, instead of that to the B.S. equation only, as done by Salpeter[14]. One will see that with our approach the calculations on the matrix elements under the Mandelstam formalism and the B.S. wave functions extracted from potential models as well, are reliable even in the cases of the transitions with quite a great momentum recoil. As previously discussed, we develop the approach with the motivation from the B_c meson physics, thus we will take the decays such as the B_c meson to $J/\psi + (\bar{l}\nu)$ as examples to illustrate it in this paper.

To calculate the exclusive weak decays of the B_c meson, one needs to evaluate the hadronic matrix elements, i.e. the weak current operator sandwiched between the initial state of the B_c meson and the concerned hadronic finial state. We restrain ourselves to evaluate them in the simplest cases, i.e. only those decays in their final state having only one hadron for semileptonic ones, but two hadrons for nonleptonic

ones. In these cases, one may attribute the problem to evaluating a matrix element of the weak current operator sandwiched by two of single-hadron states (for non-leptonic decays it is due to the factorization assumption of calculating the decay amplitude).

With the notation of a weak charged current $J_\mu = V_\mu - A_\mu$, where V_μ, A_μ are the vector and the axial vector current respectively, the matrix elements are related to the form factors[21,23] as

$$\begin{aligned} &< P(p')|V_\mu|B_c(p) >= f_+(p+p')_\mu + f_-(p-p')_\mu, \\ &< V(p',\epsilon^*)|V_\mu|B_c(p) >= ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho(p-p')^\sigma, \\ &< V(p',\epsilon^*)|A_\mu|B_c(p) >= f\epsilon^*_\mu + a_+(\epsilon^*\cdot p)(p+p')_\mu + a_-(\epsilon^*\cdot p)(p-p')_\mu. \end{aligned} \tag{1}$$

where p, p' are the momenta of the B_c and the outgoing hadron respectively, P and V denote the pseudoscalar and the vector mesons respectively, ϵ is the polarization vector of the vector meson. The form factors are functions of Lorentz invariant variable $r^2 \equiv (p-p')^2$.

In the literature one may find two kinds of approaches to calculate these form factors. One of them is BSW model[16], in which the authors calculated the form factors at the maximum recoil $r^2 = 0$ by means of the wave functions defined at the light cone system under the quark model framework, and then to extrapolate the result to all values of r^2 by assuming the form factors dominated by a proper pole of the nearest ones. The other is IGSW model[17]. The authors of Ref.[17] calculated the form factors by using the wave functions of the quark-model ("mock meson") which treats the hadrons as a nonrelativistic object. As argued by the authors, the approach is exactly valid in the limit of weak binding and at the point of zero recoil. However, in the cases with a large recoil, it is problematic. For instance, for the decay $B_c \to J/\psi + \rho$, i.e. the example we are considering, although the initial state B_c and the final state J/ψ both are of weak binding, the recoil of the decay is not small so the IGSW model may not work very well.

However, inside the B_c meson, both the $\bar{b}$ and c are heavy so may be considered being nonrelativistic objects, but their masses are comparable, thus the formalism[1−6] may not be very appropriate, at least careful examination and considerable modification are expected. Furthermore, even though their approach is not suitable for the B_c meson, to establish a link between the universal Isgur-Wise function $\xi(v \cdot v')$ and the nonrelativistic wave function of the heavy meson obtained by the potential model would be still very interesting, where v and v' are the four-velocity vectors of the meson in the initial and final states respectively.

Now let us proceed to write down the matrix element with the help of the Mandelstam formalism, so to describe the approach explicitly. It is known that the B-S equation of a fermion-antifermion bound state takes the following form:

$$(\not{p}_1 - m_1)\ \chi_p(q)\ (\not{p}_2 + m_2) = i \int \frac{d^4k}{(2\pi)^4}\ V(p,k,q)\ \chi_p(k) \tag{2}$$

where p_1, and p_2 are the momenta of the constituent particles 1 and 2 respectively. They can be expressed in terms of the total and the relative momenta p and q as

$$\begin{aligned} p_1 &= \alpha_1 p + q, \qquad \alpha_1 = \frac{m_1}{m_1 + m_2}; \\ p_2 &= \alpha_2 p - q, \qquad \alpha_2 = \frac{m_2}{m_1 + m_2}, \end{aligned} \tag{3}$$

$V(p,k,q)$ is the interaction kernel. It is well known that the B-S wave function $\chi_p(q)$ satisfies the normalization condition:

$$\int \frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4}\mathrm{tr}\{\bar{\chi}_p(q)\frac{\partial}{\partial p_0}[S_1^{-1}(p_1)S_2^{-1}(p_2)\delta^4(q-q')+V(p,q,q')]\chi_p(q')\} = 2ip_0. \tag{4}$$

As for the decays caused by the quark q_1 of the meson, according to the Mandelstam formalism[13] and the spectator mechanism shown in Fig.1, the weak current matrix element involving one hadron in the initial state and one in the final state respectively, may be expressed in terms of the B-S wave functions:

$$l^\mu(r) = i \int \frac{d^4q}{(2\pi)^4}\mathrm{tr}[\bar{\chi}'_p(q')\Gamma_1^\mu\chi_p(q)(\not{p}_2 + m_2)], \tag{5}$$

where $\chi_p(q)$, $\bar{\chi}_{p'}(q')$ are the B-S wave functions of the initial state and the final state with the total momenta p, p' and the relative momenta q, q' respectively; p_1, m_1, p_1', m_1',and p_2, m_2 are the momenta and the masses of the decay quark, the final one and the spectator respectively; Γ_1^μ is the weak interaction vertex and to the lowest order, Γ_1^μ has the form of $\gamma_\mu(1-\gamma_5)$ for the charged current.

As pointed out above, the B-S wave function $\chi_p(q)$ of the heavy quark pair system under the instantaneous approximation, can be evaluated by solving the corresponding Schrödinger equation with a QCD inspired potential. Usually, to make the nonrelativistic instantaneous approximation for the B-S equation is implied in the rest frame of the concerned bound state.

Now let us outline the the approximation but in a form, which may be used further.

The instantaneous approximation is to carry through the integration over the q_0 component for the B-S equation Eq.(2) firstly, when the kernel at the rest frame has a simple form

$$V(p,k,q) \sim V(|\mathbf{k}-\mathbf{q}|), \tag{6}$$

then as a result, the B-S equation Eq.(2) is deduced into a three-dimensional equation, i.e., the Schrödinger equation in a momentum representation[14].

However, as pointed out above, when the decays with sizable recoil are concerned, it still is not sufficient to have the relation between the wave functions in the rest frame only, because in the Mandelstam formalism Eq.(5), the start point of our approach, at least one of the B-S wave functions in a Lorentz covariant form in a moving frame is required. To pursue the purpose, we take the way to make the instantaneous approximation to the whole matrix element itself instead of only the wave functions.

As the first step, we need to divide the relative momentum q of the B-S equation into two covariant parts firstly: $q_{p\parallel}$ and $q_{p\perp}$, a parallel one and an orthogonal one

to the centre mass momentum p respectively, i.e.,

$$q^\mu = q^\mu_{p\|} + q^\mu_{p\perp} \tag{7}$$

where $q^\mu_{p\|} \equiv \frac{p\cdot q}{M_p^2}p^\mu$; $q^\mu_{p\perp} \equiv q^\mu - q^\mu_{p\|}$. Correspondingly, we have two Lorentz invariant variables:

$$\begin{aligned} q_p &= \frac{p\cdot q}{M_p}, \\ q_{p\top} &= \sqrt{q_p^2 - q^2} = \sqrt{-q^2_{p\perp}}. \end{aligned} \tag{8}$$

In the rest frame of the meson, i.e. $\mathbf{p} = 0$, they turn back to the usual component q_0 and $|\mathbf{q}|$ respectively.

Now in terms of these variables, the covariant form of the wave function can be obtained. The volume element of the relative momentum k can be written in an invariant form:

$$d^4k = dk_p k^2_{p\top} dk_{p\top} ds d\phi, \tag{9}$$

where ϕ is the azimuthal angle, $s = \dfrac{k_p q_p - k\cdot q}{k_{p\top}q_{p\top}}$. The interaction kernel can be denoted as:

$$V(|\mathbf{k}-\mathbf{q}|) = V(k_{p\perp}, s, q_{p\perp}), \tag{10}$$

which is independent of k_p and q_p. Let us introduce the notation for convenience:

$$\begin{aligned} \varphi_p(q^\mu_{p\perp}) &\equiv i\int \frac{dq_p}{2\pi}\chi_p(q^\mu_{p\|}, q^\mu_{p\perp}), \\ \eta(q^\mu_{p\perp}) &\equiv \int \frac{k^2_{p\top}dk_{p\top}ds}{(2\pi)^3}V(k_{p\perp}, s, q_{p\perp})\varphi_p(k^\mu_{p\perp}), \end{aligned} \tag{11}$$

thus the B-S equation can be rewritten in short as:

$$\chi_p(q_{p\|}, q_{p\perp}) = S_1(p_1)\eta(q_{p\perp})S_2(p_2), \tag{12}$$

where $S_1(p_1)$ and $S_2(p_2)$ are the propagators of the constituent particles and they can be decomposed as:

$$S_i(p_i) = \frac{\Lambda^+_{ip}(q_{p\perp})}{J(i)q_p + \alpha_i M - \omega_{ip} + i\epsilon} + \frac{\Lambda^-_{ip}(q_{p\perp})}{J(i)q_p + \alpha_i M + \omega_{ip} - i\epsilon}, \tag{13}$$

with

$$\begin{aligned}\omega_{ip} &= \sqrt{m_i^2 + q_{p\top}^2}\,, \\ \Lambda_{ip}^{\pm}(q_{p\perp}) &= \frac{1}{2\omega_{ip}}\left[\frac{\not{p}}{M}\omega_{ip} \pm J(i)(m_i + \not{q}_{p\perp})\right],\end{aligned} \tag{14}$$

$i = 1, 2$ and $J(i) = (-1)^{i+1}$

Here $\Lambda_{ip}^{\pm}(q_{p\perp})$ satisfies the following relations:

$$\begin{aligned}&\Lambda_{ip}^{+}(q_{p\perp}) + \Lambda_{ip}^{-}(q_{p\perp}) = \frac{\not{p}}{M}, \\ &\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\Lambda_{ip}^{\pm}(q_{p\perp}) = \Lambda_{ip}^{\pm}(q_{p\perp}), \\ &\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\Lambda_{ip}^{\mp}(q_{p\perp}) = 0.\end{aligned} \tag{15}$$

Thus, $\Lambda_{ip}^{\pm}(q_{p\perp})$ may be referred to as p-projection operators (p is the momentum of the bound state) while in the rest frame they correspond to the energy projection operators.

If defining $\varphi_p^{\pm\pm}(q_{p\perp})$ as

$$\varphi_p^{\pm\pm}(q_{p\perp}) \equiv \Lambda_{1p}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\varphi_p(q_{p\perp})\Lambda_{2p}^{\pm c}(q_{p\perp})\frac{\not{p}}{M}, \tag{16}$$

where the upper index c denotes the charge conjugation, under the notation:

$$\Lambda_{2p}^{\pm c}(q_{p\perp}) \equiv \Lambda_{2p}^{\pm}(q_{p\perp}),$$

and integrating over q_p on both sides of Eq.(12), we obtain

$$\begin{aligned}(M - \omega_{1p} - \omega_{2p})\varphi_p^{++}(q_{p\perp}) &= \Lambda_{1p}^{+}(q_{p\perp})\eta_p(q_{p\perp})\Lambda_{2p}^{+c}(q_{p\perp}), \\ (M + \omega_{1p} + \omega_{2p})\varphi_p^{--}(q_{p\perp}) &= \Lambda_{1p}^{-}(q_{p\perp})\eta_p(q_{p\perp})\Lambda_{2p}^{-c}(q_{p\perp}), \\ \varphi_p^{+-}(q_{p\perp}) = \varphi_p^{-+}(q_{p\perp}) &= 0.\end{aligned} \tag{17}$$

The normalization condition becomes into the following covariant form:

$$\int \frac{q_{p\top}^2 dq_{p\top}}{2\pi^2}\mathrm{tr}\left[\overline{\varphi}_p^{++}(q_{p\perp})\frac{\not{p}}{M}\varphi_p^{++}(q_{p\perp})\frac{\not{p}}{M} - \overline{\varphi}_p^{--}(q_{p\perp})\frac{\not{p}}{M}\varphi_p^{--}(q_{p\perp})\frac{\not{p}}{M}\right] = 2M. \tag{18}$$

Now for the usage later on, let us introduce two 3-momenta $\tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{p}}_2$,

$$\begin{aligned}\tilde{\mathbf{p}}_1 &\equiv \frac{\omega_{1p}}{M}\mathbf{p} + \mathbf{q}_{p\perp},\\ \tilde{\mathbf{p}}_2 &\equiv \frac{\omega_{2p}}{M}\mathbf{p} - \mathbf{q}_{p\perp}.\end{aligned} \tag{19}$$

In the case of the weak binding as concerned here, the wave functions of the double heavy quark systems can be constructed approximately as follows:

$$\begin{aligned}\varphi_p^{\lambda++}(q_{p\perp}) &= \sum_{ss'}\frac{1}{\sqrt{4\omega_{1p}\omega_{2p}}}u_s(\tilde{\mathbf{p}}_1)\bar{v}_{s'}(\tilde{\mathbf{p}}_2)\phi_p^+(q_{p\top})\chi^\lambda_{ss'},\\ \varphi_p^{\lambda--}(q_{p\perp}) &= \sum_{ss'}\frac{1}{\sqrt{4\omega_{1p}\omega_{2p}}}v_s(\tilde{\mathbf{p}}_1)\bar{u}_{s'}(\tilde{\mathbf{p}}_2)\phi_p^-(q_{p\top})\chi^\lambda_{ss'},\end{aligned} \tag{20}$$

where $u_s(\tilde{\mathbf{p}}_i)$, $v_{s'}(\tilde{\mathbf{p}}_i)$ (i=1,2) are the Dirac spinors of free particles with masses m_i and momenta $\tilde{\mathbf{p}}_i$; $\chi^\lambda_{ss'}$ is the Clebsch-Gordan coefficients that make s' and s couple to λ; and $\phi^\pm(q_{p\top})$ is the scalar part of the wave function.

In the present case, owing to weak binding, the $\varphi_p^{\lambda--}(q_{p\perp})$ is a small component and can be ignored. In fact, if the kernel is of scalar and/or vector, the $\varphi_p^{\lambda--}(q_{p\perp})$ is in the order of $(v/c)^4$ to $\varphi_p^{\lambda++}(q_{p\perp})$[18]. Furthermore, if ignoring the components proportional to the $q_{p\perp}$ in the spinor structure due to the nonrelativistic nature, $\varphi_p^{\lambda++}(q_{p\perp})$ can be simplified:

$$\varphi_p^{\lambda++}(q_{p\perp}) = \frac{\not{p}+M}{2\sqrt{2}M}(\alpha\gamma_5 + \beta\not{\epsilon})\phi(q_{p\top}) \tag{21}$$

where $\alpha = 1,\ \beta = 0$ for an $^1S_0$ state and $\alpha = 0,\ \beta = 1$ for a $^3S_1$ state, while the "radius" wave function $\phi(q_{p\top})$ satisfies the following Schrödinger equation:

$$\frac{q_{p\top}^2}{2\mu}\phi(q_{p\top}) + \int\frac{k_{p\top}^2 dk_{p\top}ds}{(2\pi)^3}V(s,k_{p\perp},q_{p\perp})\phi(k_{p\top}) = E\phi(q_{p\top}), \tag{22}$$

with the reduced mass $\mu = \dfrac{m_1m_2}{m_1+m_2}$ of the system.

Thus, we have established the relation in covariant form between the B-S wave function and the solution of the Schrödinger equation with the instantaneous approximation.

Now we start the procedure to make the instantaneous approximation to the whole matrix element itself. The weak current matrix element, by integrating over the q_0 component of Eq.(10) with the method of contour integration.

As asserted above, the negative energy parts of the wave functions are very small in weak binding cases so that we can ignore their contributions in the lowest order approximation. In the present case, we are concerning weak-binding bound states only, thus those contributions from the negative energy parts will be ignored. Putting Eqs.(12), (13) into Eq.(5), we have the matrix element:

$$\begin{aligned} l_\mu(r) &= \int \frac{d^4q}{(2\pi)^4}\left[\bar{\eta}'_{p'}(q'_{p'\perp})\frac{\Lambda^+_{1p'}(q'_{p'\perp})}{q'_{p'}+\alpha'_1M'-\omega_{1'p'}+i\epsilon}\Gamma_{1\mu}\right. \\ &\left.\frac{\Lambda^+_{1p}(q_{p\perp})}{q_p+\alpha_1M+\omega_{1p}+i\epsilon}\cdot\frac{\Lambda^+_{2p}(-q_{p\perp})}{q_p-\alpha_2M+\omega_{2p}-i\epsilon}\right]\eta_p(q_{p\perp}). \end{aligned} \tag{23}$$

In the bracket of the integrand, there are three poles at points a_i in the complex-q_0 plane:

$$\begin{aligned} a_1 &= -\alpha_1M+\omega_1-i\epsilon, \\ a_2 &= \alpha_2M-\omega_2+i\epsilon, \\ a'_1 &= \alpha_2M-E'\sqrt{(\mathbf{r}+\mathbf{q})^2+m'^2_1}-i\epsilon, \end{aligned} \tag{24}$$

and two branch cuts starting at the branch points:

$$q_0 \simeq m_2 \pm i\frac{m'_1}{\gamma}, \tag{25}$$

with $\gamma \equiv \dfrac{|\mathbf{r}|}{M'}$ and all the terms relevant to $\mathbf{q}$ are ignored, due to the fact $\omega'_{1p'} = \sqrt{q'^2_{p'\perp}+m'^2_1}$.

Now the problem becomes how to carry out the integration over the q_0 component on the right side of Eq.(23) or, in another words, how to treat the cuts of the integrand if contour integration method is adopted. Wereas here we will treat the branch cuts in the integrand approximately, by expanding $\omega'_{1p'}$ as follows:

$$\omega'_{1p'} = \sqrt{q'^2_{p'\perp}+m'^2_1} = m'_1+\frac{q'^2_{p'\perp}}{2m'_1}+\cdots \quad . \tag{26}$$

In a weak binding limit it is quite good approximation. According to Cauchy's theorem, the integration of a closing contour on the upper half plane of the complex-q_0 for the matrix element $l_\mu(r)$, is just that summing up all poles' residues. However, as the pole a_2 on the upper half plane is very close to the pole a_1 on the lower half plane, i.e. the distance

$$\Delta \equiv a_1 - a_2 \simeq M - m1 - m2 + \frac{\mathbf{q^2}}{2\mu_1} \tag{27}$$

is small, the value of the integration is dominated by the residue of the pole a_2 only. The contribution from the pole a' is not important i.e. may be ignored at all at the concerned accurate level.[3]. Therefore, we obtain:

$$l_\mu(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \bar{\eta}'_{p'}(q'_{p'\perp}) \frac{\Lambda^+_{1p'}(q'_{p'\perp})\Gamma_{1\mu}\Lambda^{c+}_{2p}(-q_{p\perp})\Lambda^+_{1p}(q_{p\perp})}{(q'_{p'} + \alpha'_1 M' - \omega'_{1p'})(M - \omega_{1p} - \omega_{2p})} \eta_p(q_{p\perp}) \ . \tag{28}$$

It is easy to prove the following relations

$$\begin{aligned} q'_{p'} + \alpha'_1 M' - \omega'_{1p'} &= M' - \omega'_{1p'} - \omega'_{2p'}, \\ \Lambda^{c+}_{2p}(-q_{p\perp}) &= \frac{\omega'_{2p'}}{\omega_{2p}} \Lambda^{c+}_{2p'}(-q'_{p'\perp})\gamma_0 \Lambda^{c+}_{2p}(-q_{p\perp}) \ , \end{aligned} \tag{29}$$

and with Eq.(17) from Eq.(28) we obtain the required equation:

$$l_\mu(r) \approx \int \frac{q^2_{p\top} dq_{p\top} ds}{(2\pi)^2} \mathrm{tr}\ [\ \overline{\varphi}^{++}_{p'}(q'_{p'\perp})\Gamma_\mu \varphi^{++}_p(q_{p\perp}) \frac{\not{p}}{M}\ ]\ \frac{\omega'_{2p'}}{\omega_{2p}}, \tag{30}$$

where $q'_{p'\perp}$, $\omega'_{1p'}$, $\omega'_{2p'}$, ω_{1p}, ω_{2p} are as expressed in Eqs.(8), (14) and (26) i.e.

$$\begin{aligned} \omega'_{1p'} &= \sqrt{q'^2_{p'\perp} + m'^2_1}, \\ \omega_{ip} &= \sqrt{\mathbf{q}^2 + m_i^2}, \\ \omega'_{2p'} &= \frac{E'\omega_{2p} + \mathbf{r}\cdot\mathbf{q}}{M'}, \\ q'_{p'\top} &= \sqrt{\omega'^2_{2p'} - m_2^2}. \end{aligned} \tag{31}$$

[3] In fact, some poles (even cuts) may be induced into the matrix element through $\bar{\eta}'_{p'}(q'_{p'\perp})$, however as the similar reason as here, they would not contribute so substantially as that of the pole a_2 to the final results in all the cases of weak binding.

We should note here that based on the adopted extra approximations such as Eq.(25), the Eq.(30) is valid only with not too large recoils, i.e. $\gamma \equiv \frac{|\mathrm{r}|}{M'} \leq 1$, however most of the decays of double heavy quark systems such as our concerning processes $B_c \to J/\psi + X$, $B_c \to B_s + X$ etc. satisfy the condition well.[4] Using the wave functions in the form of Eq.(20) for both of the initial state and the final state, it follows that

$$l_\mu(r) = \int \frac{q_{p\top}^2 dq_{p\top} ds}{(2\pi)^2} \left[\overline{u}_l(\tilde{\mathbf{p}'}_1)\Gamma_\mu u_m(\tilde{\mathbf{p}}_1) \right] \overline{\varphi}^+_{p'}(q'_{p'\top})\varphi^+_p(q_{p\top})\chi^\lambda_{ls'}\chi^{\lambda'}_{s'm}\left(\frac{\omega'_{2p'}}{4\omega_{2p}\omega_p\omega'_{1p'}}\right)^{\frac{1}{2}} \tag{32}$$

where

$$\begin{aligned} &\omega_{1p'} = \sqrt{q'^2_{p'\top} + m'^2_1} \\ &p_1 = (\omega_1, \mathbf{q}) \\ &p'_1 = \frac{\omega'_{1p} + \omega'_{2p}}{M'}p' - \frac{\omega_{1p} + \omega_{2p}}{M}p + p_1, \end{aligned} \tag{33}$$

and the normalization condition for the 'spectator' (the antifermion with momentum p_2 in Fig. 1),

$$\bar{v}_{s'}(p_2)\frac{\not{p}}{M}v_{l'}(p_2) = 2\omega_2\delta_{s'l'}, \tag{34}$$

has been used.

In order to compare with those results of Refs.[16,17], we compute out the form factors and it is very interesting that we may extract a similar 'universal' Isgur-Wise function at last.

There is some arbitrariness in choosing the directions of the spins of the quarks when calculating the form factors. Thus we may use it to simplify the calculations. It is convenient to take direction of the spins orthogonal to the p and p' because the spins in this direction remains unchanged if a Lorentz boost along the p' direction is taken.

[4]Here in order to show the approach emphasized in this paper briefly so as to see the inspire of the approach clearly we take the extra approximation, as matter of fact it may be weaken or improved in the practical usage.

In the Appendix of Ref.[7], a covariant formalism to calculate the creation of a pair of fermion-antifermion have been derived in the spirit of helicity amplitude[19]. We will employ the method for the present problem.

A similar formalism can be obtained for a fermion scattered by a virtue $W^{\pm}$ boson i.e. the amplitudes with possible spin directions read as

$$\begin{aligned} M^{\mu}_{1,2} &= L_{+}\frac{1}{2}\mathrm{tr}\left[(\not{p}'_1 + m'_1)\frac{1 \pm \gamma_5 \not{k}_1}{2}(\not{p}_1 + m_1)\Gamma^{\mu}\right] \\ M^{\mu}_{3,4} &= L_{-}\frac{1}{2}\mathrm{tr}\left[(\not{p}'_1 + m'_1)\gamma_5\frac{1 \pm \gamma_5 \not{k}_1}{2}(\not{p}_1 + m_1)\Gamma^{\mu}\right] \end{aligned} \tag{35}$$

where

$$Ł_{\pm} = [\frac{1}{2}(p_1 \cdot p'_1 \pm m_1 m'_1)]^{-\frac{1}{2}}, \tag{36}$$

and k_1 is an auxiliary and space-like vector ($k_1^2 = -1$), which is orthogonal to the initial momentum p_1 and the final one p'_1. It is easy to see that the first equation of Eq.(25) describes the spin nonflip amplitude while the second describes a flip one. Both fermions are fully polarized along the $\pm k_1$ directions. It should be noted here that in these formulae the relative phases of the spinors among those states with different polarizations have been fixed.

To calculate the form factors from a current matrix element, we need to construct the spin wave functions of the individual quarks into definite spin ones to describe the initial and the final states of the matrix element respectively. In general, for an $^1S_0$ state

$$\chi_{ss'} = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \tag{37}$$

and for an $^3S_1$ state, the spin structure corresponding to the three possible independent polarizations are

$$\begin{aligned} \chi^{k_1}_{ss'} &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \\ \chi^{k_2}_{ss'} &= \frac{1}{\sqrt{2}}(\uparrow\uparrow + \downarrow\downarrow), \\ \chi^{k_3}_{ss'} &= \frac{i}{\sqrt{2}}(\uparrow\uparrow - \downarrow\downarrow), \end{aligned} \tag{38}$$

where three vectors k_1, k_2, and k_3 orthogonal to each other, are used to denote the polarization directions of the ${}^3S_1$ state.

Thus, for a transition $P \to P' + X$, the amplitude reads

$$M_0^\mu = L_+ \frac{1}{2} \mathrm{tr}[\ (\not{p}_1' + m_1')\gamma_5\gamma_5(\not{p}_1 + m_1)\Gamma^\mu\ ], \tag{39}$$

and, for a transition $P \to V + X$, the corresponding amplitudes read

$$\begin{aligned} M_1^\mu &= L_+ \frac{1}{2} \mathrm{tr}[\ (\not{p}_1' + m_1')\gamma_5 \not{k}_1 (\not{p}_1 + m_1)\Gamma^\mu\ ], \\ M_2^\mu &= L_- \frac{1}{2} \mathrm{tr}[\ (\not{p}_1' + m_1') \not{k}_1 (\not{p}_1 + m_1)\Gamma^\mu\ ], \\ M_3^\mu &= L_- \frac{1}{2} \mathrm{tr}[\ (\not{p}_1' + m_1')\gamma_5 (\not{p}_1 + m_1)\Gamma^\mu\ ], \end{aligned} \tag{40}$$

where M_1, M_2 and M_3 correspond to those of various projections of the polarizations of the final states. In fact, if letting

$$\begin{aligned} k_2^\mu &= \frac{1}{2} L_+ L_- \epsilon^{\mu\nu\rho\sigma} p_{1\nu}' k_{1\rho} p_\sigma, \\ k_3^\mu &= \frac{L_+ L_-}{2M'} [(p' \cdot p_1') p_1^\mu - (p' \cdot p) p_1^{'\mu}] \end{aligned} \tag{41}$$

correspond to the polarizations of M_2 and M_3, the amplitudes of the $P \to V + X$ transition can be written down in a compact form:

$$M^\mu = L_+ \frac{1}{2} \mathrm{tr}[\ (\not{p}_1' + m_1')\gamma_5 \not{\epsilon}'(\not{p}_1 + m_1)\Gamma^\mu\ ], \tag{42}$$

where

$$\begin{aligned} \epsilon^{'\mu} &= \epsilon^\mu - \frac{C\dot{(}\epsilon \cdot p)}{p^2 - \dfrac{(p \cdot p')}{m'^2}} p_{p'\perp}^\mu, \\ C &= \frac{L_+}{\sqrt{\dfrac{(p' \cdot p_1)(p' \cdot p_1')}{M'^2} - \dfrac{1}{L_-^2}}} - 1, \\ p_{p'\perp} &= p - \frac{(p \cdot p')}{M'^2} p'. \end{aligned} \tag{43}$$

After a straightforward calculation, the form factors may be formulated:

$$\begin{aligned} f_{\pm} &= \xi\left[\frac{1}{M}(1-\frac{m_2}{m_1'})\pm\frac{\omega_1'+\omega_2'}{M'm_1'}\right], \\ g &= \xi\frac{\omega_1'+\omega_2'}{MM'm_1'}, \\ f &= \xi\left(\frac{(p\cdot p_1')}{Mm_1'}+1\right), \\ a_{\pm} &= \xi\left[\frac{2m_2}{M^2m_1'}+\delta\mp\left(\frac{\omega_1'+\omega_2'}{MM'm_1'}+\frac{(p\cdot p')}{M'^2}\delta\right)\right], \end{aligned} \tag{44}$$

where

$$\delta = -\frac{C\dot{(}1+\frac{(p\cdot p_1')}{Mm_1'})}{p^2-\frac{(p\cdot p')^2}{M'^2}}. \tag{45}$$

In the case of the zero recoil vicinity ($\mathbf{r}\to 0$)

$$\delta \to -\frac{m_2}{M^2m_1'}, \tag{46}$$

while the "common" factor, which may be considered as the 'universal' Isgur-Wise function, is written in the frame of the initial meson at rest ($\mathbf{p}=0$):

$$\xi = \left(\frac{2\omega_2'm_1^2m_1'^2}{((p_1\cdot p_1')+m_1m_1')\omega_1\omega_1'\omega_2}\right)^{\frac{1}{2}}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\phi_{p'}^{'*}(q_{p'\top}')\cdot\phi_p(|\mathbf{q}|), \tag{47}$$

where $\phi_{p'}'^{*}(q_{p'\top}')$ and $\phi_p(|\mathbf{q}|)$ correspond to the radius parts of the wave functions of the meson in the initial state and the one in the final state, respectively. Being in a covariant form and in the C.M.S. one, they may be obtained by solving the Schrödinger equation Eq.(22) and its specific one in its center mass system respectively, as long as the QCD inspired potential is transformed into the corresponding form in the equation. We should note i). Of the approximations, the instantaneous one on the whole matrix element, is essential in the approach, and we think it is relible to the purposes due to the facts that we restrict to apply this approach only

to the weak binding systems and the approximation only for the B-S equation, is proven to work well in many cases as long as a weak binding system is concerned[14]. ii). The wave functions obtained by the present way are more reliable than those by other ways, because the adopted potential is proven to work well for heavy quark systems, although the 'universal' function ξ being the overlap integration of the wave functions, does not sensitive to the specific radius wave functions very much, especially when the momentum recoil is not great it is contralled by the normalization condition of the wave functions, i.e. the overlap integration is approaching to the normalization when the recoil is approaching to zero and the reduced mass of the final state is approaching to that of the initial one in the meantime. iii). When carrying through the trace for γ matrices in the Eqs.(39) and (42) so as to reach to the form factos, the contributions from $(\vec{\gamma}\cdot\mathbf{q})$ in the terms $\not{p}_1'$ and $\not{p}_1$, have been ignored savely in the considered accuracy, because they are small in the case of weak binding, and when carrying through the integration Eq.(47), of the integrand all terms proportional to $\mathbf{q}$ in odd power will not contribute at all (only even power terms contribute).

The proposed approach to calculate the weak decay matrix elements, so as to the form factors is completed. It is expected that the approach is available as long as the mesons in initial and final states are of weak binding. It is interesting to compare ours with that of IGSW model[17]. In their model, the authors calculated the form factors by using the Gaussian-type wave functions, of which the parameters are determined by the variational method. It is easy to see that in the case of weak binding and at zero recoil vicinity, the formalism of our approach is consistent with theirs except a tiny difference in the formulation for the overlap integration of the wave functions[17][20]. However still in the case of weak binding but with a large recoil, there are two remarkable deviations between the two approaches. One deviation comes from the difference in the spin structure of the wave functions. The

other from the arguments in the wave function integrand. For instance, the function corresponding to the ξ in IGSW model is $\sqrt{\frac{M_{B_c}}{M'}}F_3$ and reads[17]:

$$\xi_{IGSW} = \left(\frac{2\beta\beta'}{\beta^2+\beta'^2}\right)^{\frac{3}{2}} \exp^{-\frac{m_2^2}{2\tilde{\mathrm{M}}\tilde{\mathrm{M}}'}\frac{t_m - t}{\kappa^2(\beta^2+\beta'^2)}} \quad , \tag{48}$$

where $t_m = (M - M')^2$; $\tilde{\mathrm{M}}$ and $\tilde{\mathrm{M}}'$ are the masses of the "mock meson"[17]; β and β' are the variational parameters for the initial and the final states respectively; m_2 is the mass of the 'spectator' and κ is a parameter introduced by hand. In the IGSW model, κ is adjusted to be 0.7 by fitting the π electromagnetism form factor and the authors of Ref.[17] regarded it as relativistic corrections due to a large recoil. However in our approach it is different i.e. all factors come into the formula automatically. The interesting thing is that occasionally the numerical calculation shows that the function of ξ obtained in our approach, is very close to that of the IGSW model with $\kappa = 0.7$. To show the fact, we present the corresponding ξ functions of $B_c \to J/\psi + X$ and $B_c \to B_s + X$ in Fig.2 and Fig.3 respectively. The dotted line represents the ξ function obtained by the Gaussian wave functions and with the original formalism of Ref.[17] and $\kappa = 1.$ as well. The dashed line represents that obtained by the wave functions from solving the Schrödinger equation of the potential model[21] with the same formalism. The dotted-dashed line represents that of the IGSW model but with $\kappa = 0.7$. Note that the parameters $\beta_{B_c} = 0.88$ and $\beta_{J/\Psi} = 0.65$ which were obtained by the authors of Ref.[22] are used here for the dotted, dashed and dotted-dashed three curves. The solid line represents the ξ function achieved by our approach with the wave functions obtained by the potential model[23]. It is easy to see from the Fig.2 that the result of Eq.(47), is very close to that of IGSW model with $\kappa = 0.7$. It means that our results involve reasonable effects automatically. In order to have more comparison we also show the ξ function for $B_c \to B_s + X$ in Fig.3 and the meaning of each line is the same as in Fig.2, although we do not expect that our approach is so suitable as that for the formal

decays $B_c \to J/\psi + X$, due to the s quark is not so heavy.

It is also very interesting to see the behavior in the limit when the quark mass is approaching to infinite because it will let us see the relation between the form factors obtained here and the universal Isgur-Wise function. At the limit of

$$m_2 << m_1, m_1' \quad \text{and} \quad m_1, m_1' \to \infty, \tag{49}$$

the formulae of Eqs. (44) – (47) reproduce those of the Isgur-Wise formalism[1−6] for the form factors. In fact, in the limitation of Eq.(49) and from Eq.(33) and Eq.(43), we have

$$\begin{aligned} p_1 &\to m_1 \cdot v, \\ p_1' &\to m_1' \cdot v', \\ \epsilon' &\to \epsilon'. \end{aligned} \tag{50}$$

Hence the Eq.(42) can be rewritten as

$$l_\mu(r) = \xi(v \cdot v') \mathrm{tr}\left[(1 + \not{v}')(\alpha + \beta\gamma_5 \not{\epsilon})(1 + \not{v})\right], \tag{51}$$

where $\xi(v \cdot v')$ is just the universal Isgur-Wise function and has the following form:

$$\xi(v \cdot v') = \frac{\sqrt{2v \cdot v'}}{\sqrt{1 + v \cdot v'}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \phi'^*(q'_{p'\top}) \phi(|\mathrm{q}|). \tag{52}$$

Thus the Eq.(51) and Eq.(52) reproduce those of the Isgur-Wise's formalism in the infinite heavy limit[1−3], so a link between the Isgur-Wise function and the non-relativistic wave function overlap integration has been established. A factor of $[0.5(1 + v \cdot v')]^{-\frac{1}{2}}$ has been derived by Bjorken[24] by using the Cabibbo-Radicati sum rule[25], however Here in our formalism it automatically appears in Eq.(52).

In Ref.[9], we have applied the approach to calculate the weak decays of the B_c systematically. It is no doubt that when the experimental study of B_c meson has fruitful results, our approach will receive serious tests. We would emphasize here that for the decay modes $B_c \to J/\Psi(\eta_c) + X$, which might be used as the most

typical channels to identify the B_c meson later on in experiments, the results from our approach are the most reliable ones, as in the calculation of the form factors the adopted nonrelativistic wave functions obtained from the potential model are tested well in addition to our approach itself is suitable them. Although our predictions to the processes are very close to those of IGSW model with $\kappa = 0.7$[20], there exists some deviations in numerical results for the processes $B_c \to B_s + X$, i,e, our predictions to the laters are larger than those of Ref.[20] both for semileptonic decays and nonleptonic decays. The deviations may be understood as follows: although the ξ function for the modes is smaller than that of the IGSW model as shown in Fig.3, the form factors gain an enhancement from the spinor factor as shown in Eq.(39) and Eq.(42).

We should point out that the approach has quite wide potential applications, as long as the binding is weak . For example, the approach may be applied to those decays between the baryons which contain two heavy quarks in addition to the B_c meson's by means of a heavy diquark picture, and we will describe it in Ref.[26].

As stated above, when taking the instantaneous approximation to the whole matrix element by doing the integration of q_0 on Eq.(23), an extra approximation has been adopted i.e. to take the expansion Eq.(26), thus the branch points are alternated. The extra approximation set down one more restraint

$$\frac{q_{p'\perp}^{'2}}{m_1^{'2}} \ll O(1)$$

i.e. the momentum recoil cannot be too great. If treating the branch points in the integrand by an accurate method i.e. making cuts, in principle, we may integrate out q_0 exactly. However, the potential models only offer us the numerical solutions owing to the complicated concrete potential, thus we have to deal with an integration not analytically throughout, thus we cannot guarantee results so accurate by numerically integrating a counter with cuts, finally as a result the accurateness is lost too. We would conclude that the extra approximation may be weaken, but not be avoid at

all[26]. The experimental information on various decays is desired very much now, as we are at the position, that the approach remain to be tested and improved guiding by the information. In the near future the experiments especially on the B_c decays are very important. Having the information, we will learn much on the decay mechanism as will as on the approach.

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Figure Captions

1. Feynman diagram corresponding to the weak current matrix element sandwiched by the B_c meson state as the initial state and a single particle state of the concerned final state.

2. The ξ function for $B_c \to J/\psi + X$. The dashed line: the ξ function of IGSW model with $\kappa = 1.$; the dotted-dashed line: the ξ function obtained by the formalism of Ref.[18] with the wave function solved by potential I[12]; the doted line: IGSW model with $\kappa = 0.7$; the solid line: the ξ function obtained by our approach.

3. The ξ function for $B_c \to B_s + X$. The meaning of each type line is the same as that in Fig. 3.